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Electromagnetic Susceptibility Measurements with a TEM Cell

F.R. Hunt

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ELECTROMAGNETIC SUSCEPTIBILITY MEASUREMENTS WITH A TEM CELL

F.R. Hunt

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ABSTRACT

A large Crawford (TEM) cell (2.74 m × 2.74 m × 5.48 m) was built by the Electrical Engineering Division of NRCC in 1982. Measurements of the electric fields in the empty cell fell within experimental error of the anticipated theoretical values. Three methods of determining the fields at equipment under test (EUT) in the center of the chamber were investigated - Time Domain Reflectometry (TDR) and the measurement of reflection coefficients (both at the input to the cell) and a monitor probe mounted in the unused half of the cell opposite the EUT. The latter method was the only one that would provide the required accuracy over the frequency range of interest. The methods for measuring EUT susceptibility and antenna factors for small antennas (<1.35 m in the longest dimension) are outlined. It is also shown that Laplace's Equation (Relaxation Method) can lead to easy solutions of the electric fields in empty and occupied cells.

RÉSUMÉ

Une grande cellule de Crawford (MET) de 2,74 m × 2,74 m × 5,48 m a été contruite par la Division de génie électrique du CNRC en 1982. Les mesures des champs électriques à l'interieur de la cellule vide approchaient les valeurs théoriques prévues avec des écarts inférieurs à l'erreur expérimentale. Trois méthodes de calcul des champs autour de l'équipement à l'essai (EAE), au centre de la chambre, ont été examinées : la réflectométrie des domaines temporels (RDT) et la mesure des coefficients de réflexion (toutes deux à l'entrée de la cellule) et une sonde de contrôle montée dans la moitié inutilisée, à l'opposé de l'EAE. Cette dernière méthode a été la seule à avoir la précision requise dans la gamme de fréquences d'intérêt. Les méthodes de mesure de la susceptibilité de l'EAE et des paramètres des petites antennes (dimension la plus grande <1,35 m) sont décrites. On montre aussi que l'équation de Laplace (méthode de relaxation) peut donner de solutions rapides pour le champ électrique dans la cellule, qu'elle soit vide ou occupée.

ELECTROMAGNETIC SUSCEPTIBILITY MEASUREMENTS WITH A TEM CELL

F.R. Hunt

INTRODUCTION

In 1981, the Electromagnetic Engineering Section embarked on a project to aid Canadian industry through the Canadian Standards Association in providing laboratory data on Electromagnetic Compatibility studies. A large Crawford (TEM) cell was completed by July 1982. This cell can be used to measure susceptibility of electronic components to rf fields over a frequency range from DC to 65 MHz. It can also be used to determine the antenna factors of simple antennas (monopoles, dipoles, and loops) over the same frequency range provided that they fit into the cell. The cell can also be used in determining the strength of unwanted radiation from electronic equipment. However, the latter use will not be described in this report.

CALIBRATION OF THE CELL

The TEM cell was built following the outline of one constructed by the National Bureau of Standards (1). The dimensions of the cell are shown in Fig. 1. The NRCC's cell differed from that of the NBS in that the septum was placed vertically as this appeared to offer better symmetry, i.e., the vertical supports for the septum were symmetrical to both halves of the cell. To provide lighting for the interior of the cell, the outer walls were made of copper screening. The costly end connectors used by NBS were replaced with the more common Type N.

The TEM cell was designed with an input impedance of 50 Ω in order to match existing components. The calculated cell cut-off frequency (2) for the TE_{10} is 66.3 MHz while the upper useful frequency limit is 75.7 MHz. Since 70 MHz was a convenient upper limit to use with the electronic equipment employed, most automatic measurements were terminated at this point. It is possible to make measurements up to 100 MHz but special techniques and calibration of the cell are required.

In order to determine the fields in the cell, the technique using Laplace's Equations described in Appendix A was employed. A convenient three-dimensional grid on 11.4 cm (4.5 in.) centers was used for one octant of the cell. The grid extended into the tapered end although it was known that the results would not be accurate in this region. The reason for the

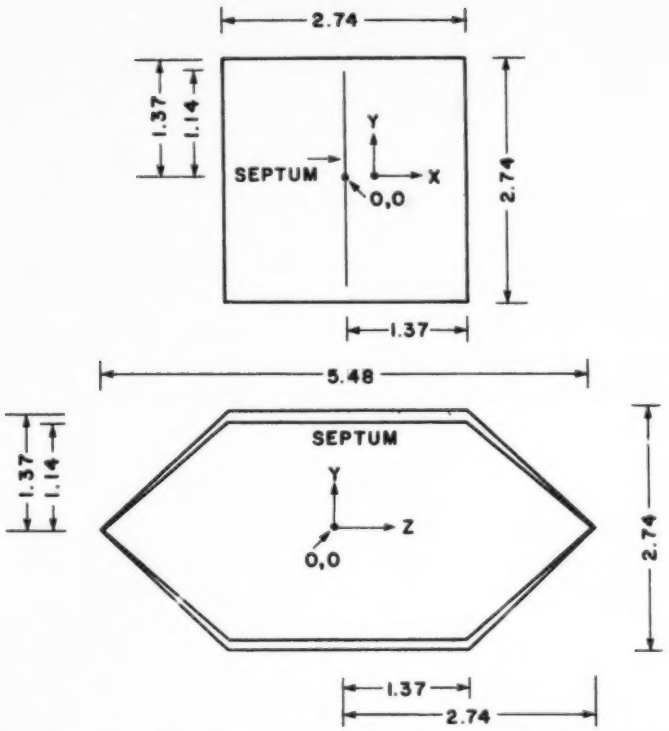

Figure 1. NRCC's large TEM cell, dimensions in metres

inaccuracy is that the taper of the outer wall is 45°, therefore the taper of the edge of the septum cannot be fitted exactly to the cubical grid. However, the inaccuracy at the large end of taper will not significantly affect the accuracy in the central region of the cell. The process was continued until the differences in potentials between successive iterations were less than 0.1%. The electric field potential in all three axes and the total were then calculated. The results are shown in Figs. 2 and 3 for two perpendicular cuts of the cell. In each case, the total field strength in volts/metre for 1 V into the cell is plotted versus the Y or Z coordinate, respectively. The curves plotted are for constant X where X is the distance from the septum. It is seen that near the center of the chamber the differences in field strength over the width of the cell are very small.

To experimentally check these figures, a 10-cm E-probe was constructed based on Greene's design (3). It was connected to a Fluke High Impedance Input DC voltmeter and calibrated in the manner described in Kashyap and Hunt (4). A cubical grid on 18 in. centers was set up inside one half of the cell and the electric field intensities measured at these points at 5, 10, 20, and 40 MHz. The results for each point were averaged and plotted as "+s" on Figs. 2 and 3. If the calculated and measured values agreed,

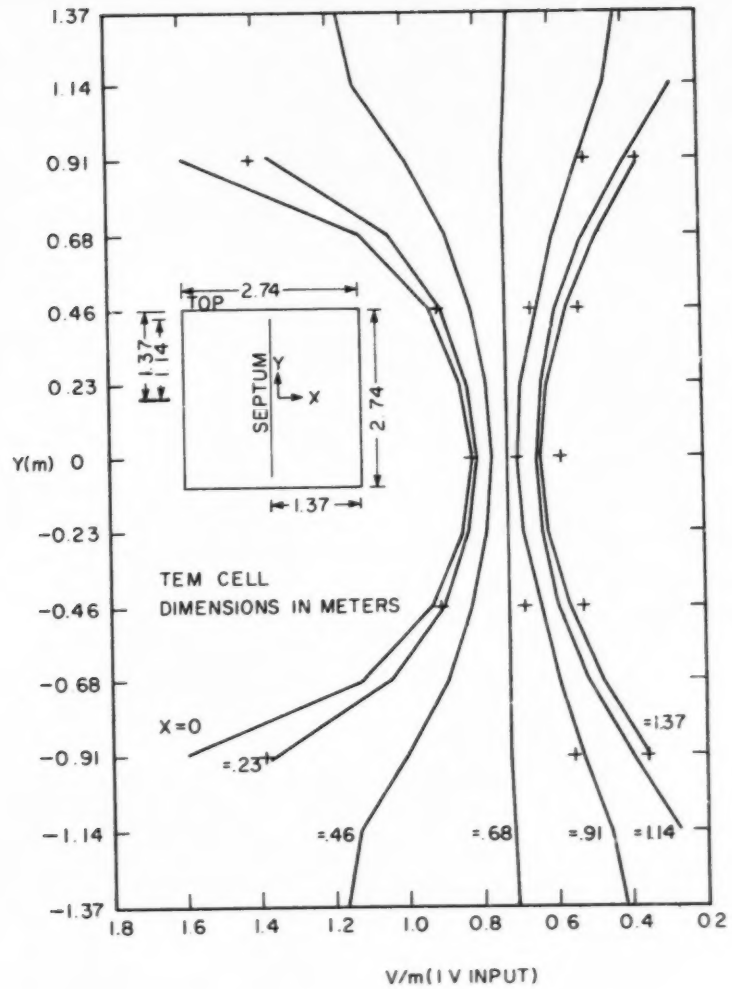

Figure 2. Total field strength at z = 0.

the "+s" should be along the lines X = 0.23, 0.68, and 1.14 m. As can be seen, most of the deviations between the two values are very small. The first major reason for deviations is that the E-probe measurement procedure was subject to temporal drift because of the high impedance employed. The second major reason was that the E-probe was basically a dipole whose axis was aligned with the X-axis of the cell. Hence, it measured essentially the X-field and not the Y and Z fields. This probably accounts for the maximum 3-dB deviations found at the X = 0.68, Y = 0.91 points in Fig. 2. At these two points, theory predicts a sizeable Y-field and a moderate X-field unlike the other points in the chamber. Because of the close agreement with theoretical values and the known errors in the field

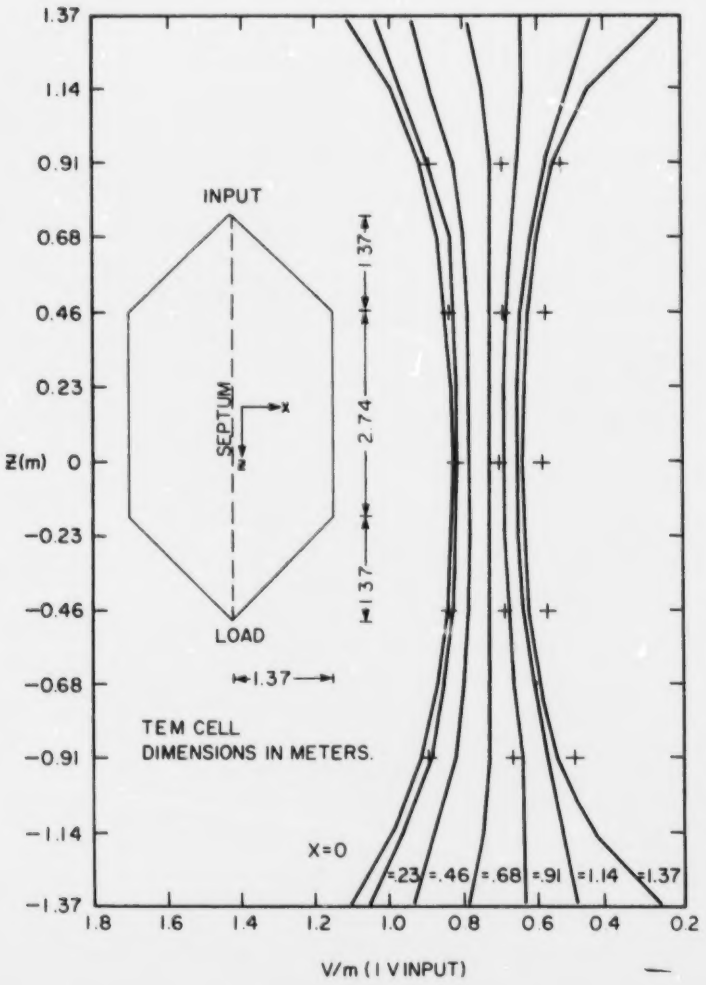

Figure 3. Total field strength at y = 0.

measurements, the theoretical field values were used in subsequent measurements.

Figure 4 depicts the electric field for various values of X at the center of the cell, i.e., y = z = 0. In using the chamber, the field strength at the X-position of the equipment under test (EUT) is used as the field at any point on the EUT. Figure 4 was compiled using the 3-D application of Laplace's Equations. A 2-D application in the Z = 0 plane produces a curve that disagrees by as much as 0.2 dB near the wall or septum. Therefore, in practice, the calculated field in Fig. 4 was used for experimental purposes.

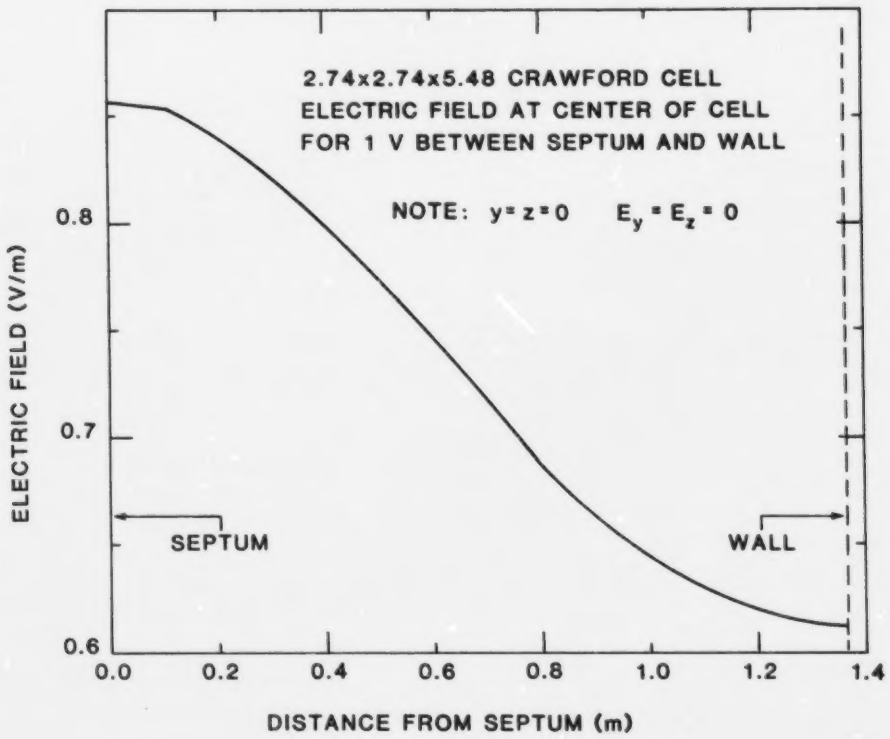

Figure 4. Electric field at centre of cell.

Existing FCC regulations for EMC testing of equipment specify that the maximum deviation in field strength between any two points in the empty volume to be later occupied by the EUT must be less than 2 dB. Examination of the 3-D field patterns shows that this volume can be represented by a cube, the length of whose sides is approximately one third the distance between the septum and wall of the cell. In the case of NRCC's cell this would be 45 cm. However, useful measurements can be made if this limit is exceeded, if proper precautions are taken.

EMC MEASUREMENT EQUIPMENT

The interconnections between various components for automatic EMC measurements in the 2 to 70 MHz region are shown in Fig. 5. The HP85 microcomputer is the controller and data recorder for the system. The FRH281178 Interface Unit built by NRCC controls several items of equipment that cannot be directly controlled by the HP85 through the HP1B line. It consists of an 8080 microprocessor and associated IC's. An internal program allows it to communicate with the HP85 through a GP1D line. It controls a relay

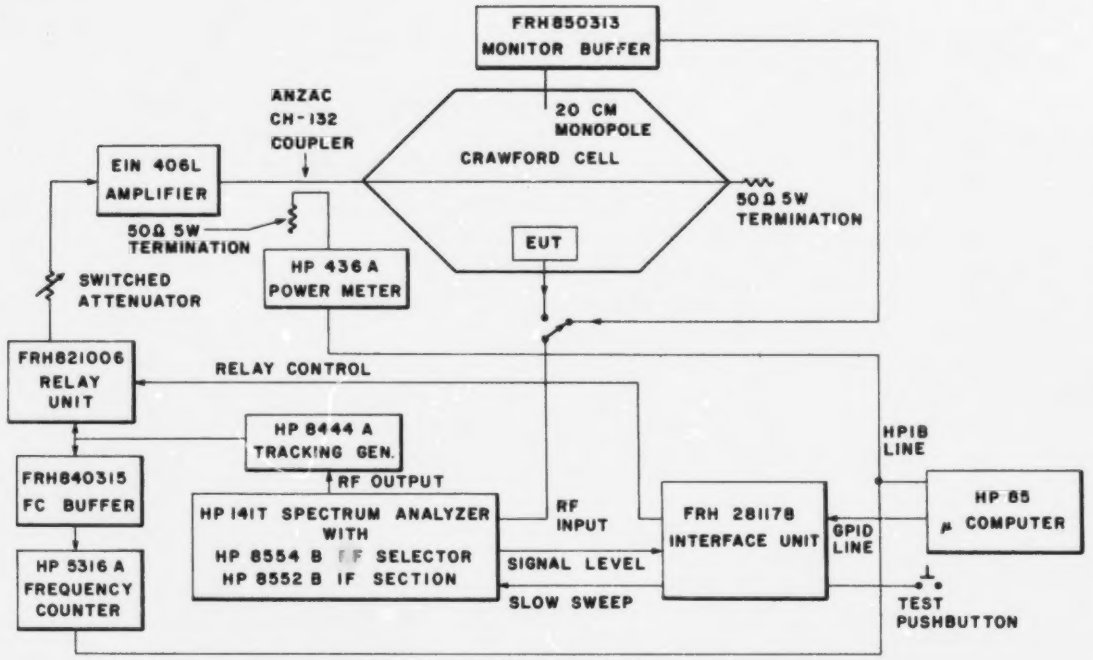

Figure 5. TEM cell interconnections: 2-70 MHz testing; HP, Hewlett Packard equipment; FRH, NRC-built equipment.

unit which grounds the input to the power amplifier to the cell while the power meter monitoring the input to the cell is being zero set prior to a measurement. It also provides an external sweep of 256 steps to the spectrum analyzer. The vertical output of the spectrum analyzer which corresponds to the signal level at the rf input to the spectrum analyzer is also digitized in the interface unit and fed to the HP85. Lastly, a signal for the HP85 is generated when a push-button switch is operated. In an EMC measurement, this push-button signal indicates that the equipment under test has shown deterioration of performance.

The rf system consists of the spectrum analyzer driving a tracking generator which produces a 0 dBm signal at the frequency to which the receiver portion of the analyzer is tuned. The output of the tracking generator is connected to a frequency counter via a buffer. The counter provides an accurate indication of frequency to the HP85 at each step of the analyzer. The output of the tracking generator is also fed to the relay unit mentioned previously. The output of the power amplifier is adjusted to provide the required electric field level in the cell by means of the switched attenuator at its input. The power amplifier has a nominal 5-W maximum output which will provide a maximum voltage on the septum of 15 V. In an empty cell, this corresponds to a field of 11 V/m in the

center of one half the cell. The power meter monitors the input power to the cell through a 19 dB directional coupler whose directivity is better than 50 dB. In the frequency sweep 2 to 70 MHz, the output power of the rf generator chain varies ±1 dB about its mean value or 11%. This is within the tolerances of the equipment. However, because of this variation, the voltage input to the cell is measured by means of the power meter at each step in frequency.

In most cases, the voltage measured at the input to the cell is sufficient in calculating the field in the cell if the longest dimension of the EUT is less than 0.46 m. However, if the longest dimension exceeds this value, the EUT reacts with the cell at unpredictable frequencies and the voltage on the center conductor opposite the EUT is uncertain. This problem will be dealt with in more detail in subsequent paragraphs. To bypass the problem, a 20-cm monopole probe with a buffer amplifier has been installed in the unused half of the cell directly opposite where the EUT is placed. Its output is normally connected to the rf input of the spectrum analyzer. The probe is calibrated in terms of voltage on the center conductor and this is the voltage used in determining the field strength at the EUT. In some cases, such as calibrating a small receiving antenna, the EUT is connected to rf input of the spectrum analyzer.

TIME DOMAIN REFLECTOMETER USE

Crawford and Workmann (2) suggest using a Time Domain Reflectometer to measure the disturbed impedance of the cell when the EUT is inserted. This value is used to determine the voltage on the center conductor. However, it is pointed out in the reference that if the wavelength of the test signal is not significantly greater than the cell length ($\lambda \geq 10L$), the TDR method produces errors. This agrees with the results obtained at the NRCC.

A 1 m × 0.25 m × 0.25 m box was constructed. At the center of the square end, a 15-cm length of wire was fastened to a connector fitted in the center of the wall of the cell. This connector allowed various loads to be placed on the box ranging from a short circuit to a very high impedance. The box was situated with its long 1-m axis parallel to the X axis of the cell. Subsequent measurements showed that the box could be replaced by a series circuit, Fig. 6, with admittance Y_D where G = 0.1 mhos, L = 2 μH, and C = 10 pF are approximate values.

Solving the equations given by Oliver (5), the reflected signal at the input to the reflectometer is given by

$$[1] \qquad h(t^1) = 1 - 0.057 \exp(-3.5 \times 10^7 t^1) \sin(2.21 \times 10^8 t^1)$$

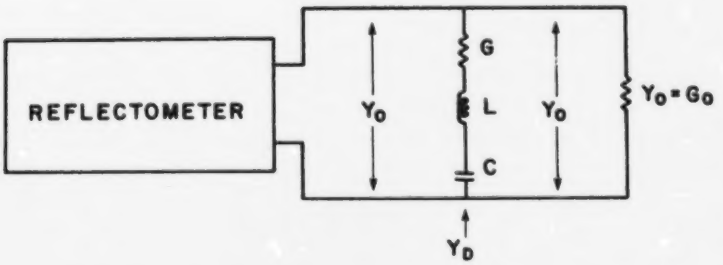

Figure 6. Reflectometer and the TEM cell.

where $t^1 = 0$ is the time taken for the TDR output pulse to travel the distance to and from the center of the cell, i.e., at the point where Y_D is connected. Equation [1] is plotted in Fig. 7 with t^1 at 2 units from the left. It should be noted that the frequency of the sine term is approximately 35.2 MHz, which was subsequently found to be one of the frequencies at which the box produced an effect in the cell. Figures 8 and 9 show photographs of a TDR screen without and with the box (EUT) inserted in the cell. Here t^1 is approximately 3 major units from the left. In the empty cell, the major excursions to the right of center are reflections at the remote end of the cell due to less than optimum connectors, loads, etc. The reflection with the grounded box are similar to the theoretical curve of Fig. 7. However, the other miscellaneous reflections do not permit an accurate determination of the reflection coefficient "ρ".

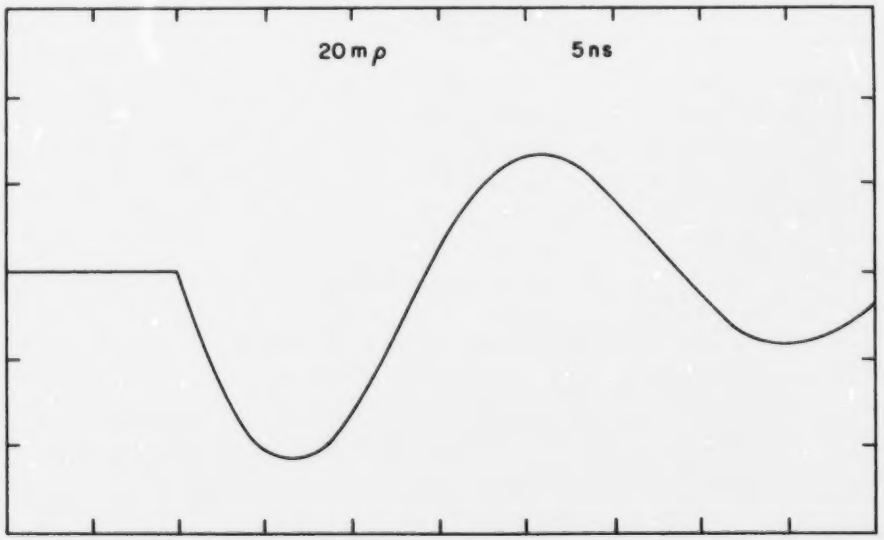

Figure 7. Theoretical reflectometer return, grounded 1 m × 0.25 m × 0.25 m box.

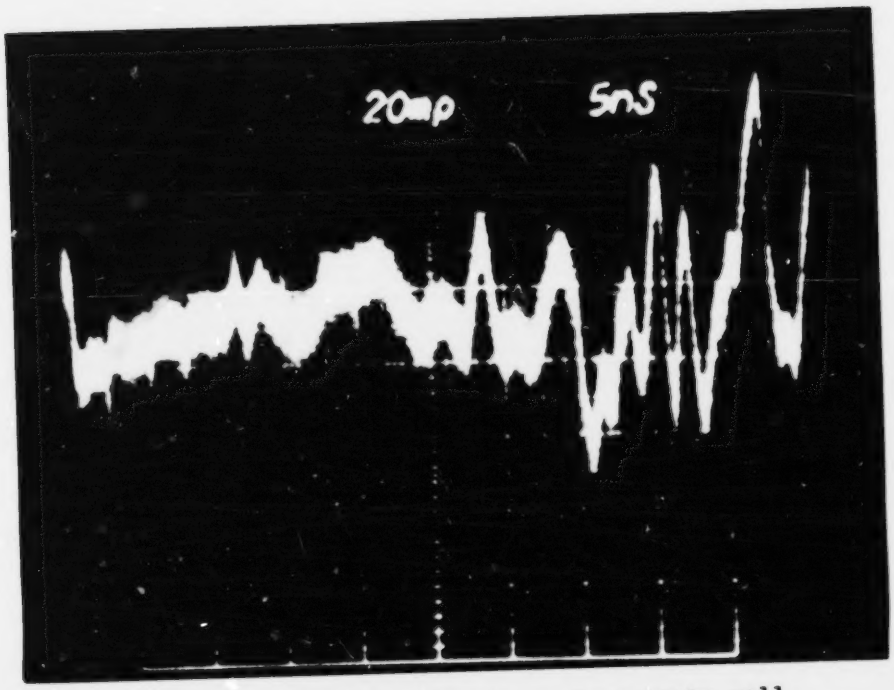

Figure 8. Reflectometer return from empty cell.

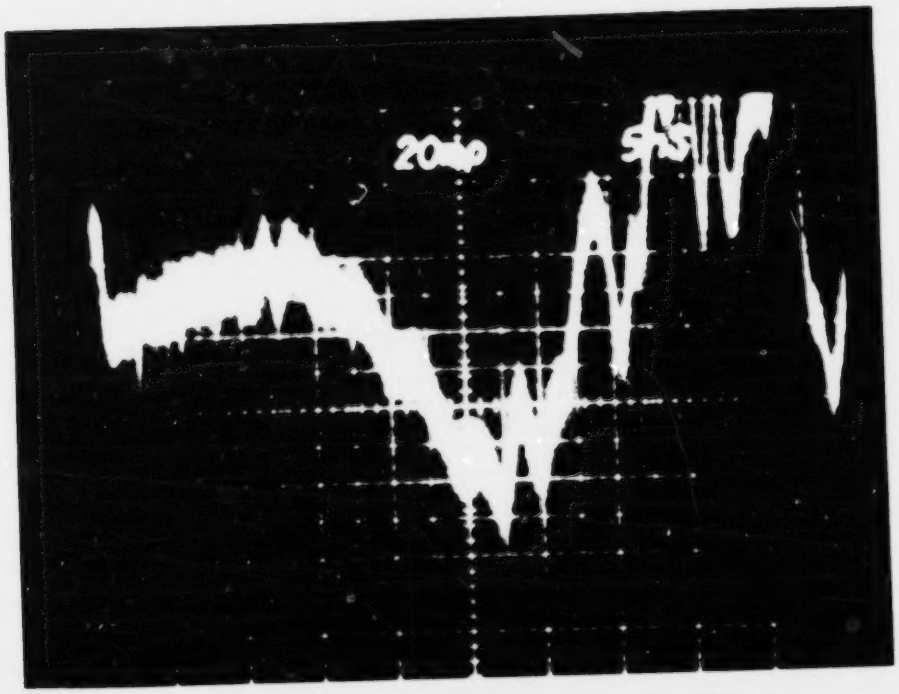

Figure 9. Reflectometer return from grounded box.

REFLECTION COEFFICIENTS

As noted in the previous paragraph, it was difficult to determine from the reflectometer display the frequencies at which an EUT would interact with the Crawford cell. The bidirectional coupler at the input to the cell permitted the measurement of both the reflected and transmitted powers to the cell. To measure the reflected power, the signal input of the Spectrum Analyzer was connected to the coupler in place of the 50-Ω load. The 1 m × 0.25 m × 0.25 m box used in the reflectometer tests was also used here. The transmission line equations were solved for the voltage input to the cell as shown in Fig. 10. Since measurements were made with the connector input to the box, grounded, terminated in 50 Ω, and open circuit, an additional two components R_p and C_p have been added to those for the box in Fig. 6. Again frequency sweeps were carried out in the 2-70 MHz range. The results are shown in Fig. 11, where the Y-axis shows the power ratio of the transmitted power to reflected power.

In the empty cell, it will be noted that the reflected power is always less than the transmitted power by at least 30 dB. If the cell was properly terminated one would expect a constant ratio of 50 dB since this is the directivity of the coupler. However, it was seen in the section on the reflectometer that the reflections from the remote end were not perfect.

In the case of the grounded box (b), a significant reflection occurs at about 35 MHz, which corresponds with the results obtained with the reflectometer. When the box is terminated with 50 Ω, significant reflections of about 10 dB occur at all frequencies above 10 MHz. When the box is unterminated, i.e., $R_p = \infty$, a significant reflection occurs at about 58 MHz. Solutions of the transmission equations, using the values shown, provide theoretical curves similar to the experimental curves. The one significant difference occurs in the empty cell where no reflections occur in the ideal cell.

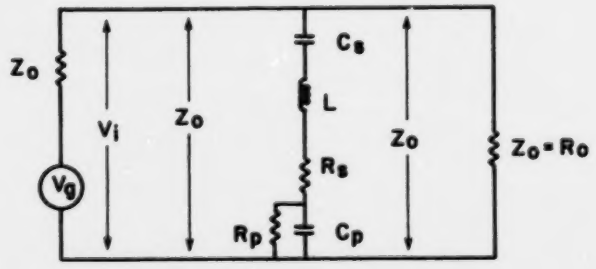

Figure 10. Reflection factors and the TEM cell: C_s = 10 pFd, C_p = 7.5 pFd, R_0 = 0 to ∞ Ω, L = 2 μH, R_p = 10 Ω.

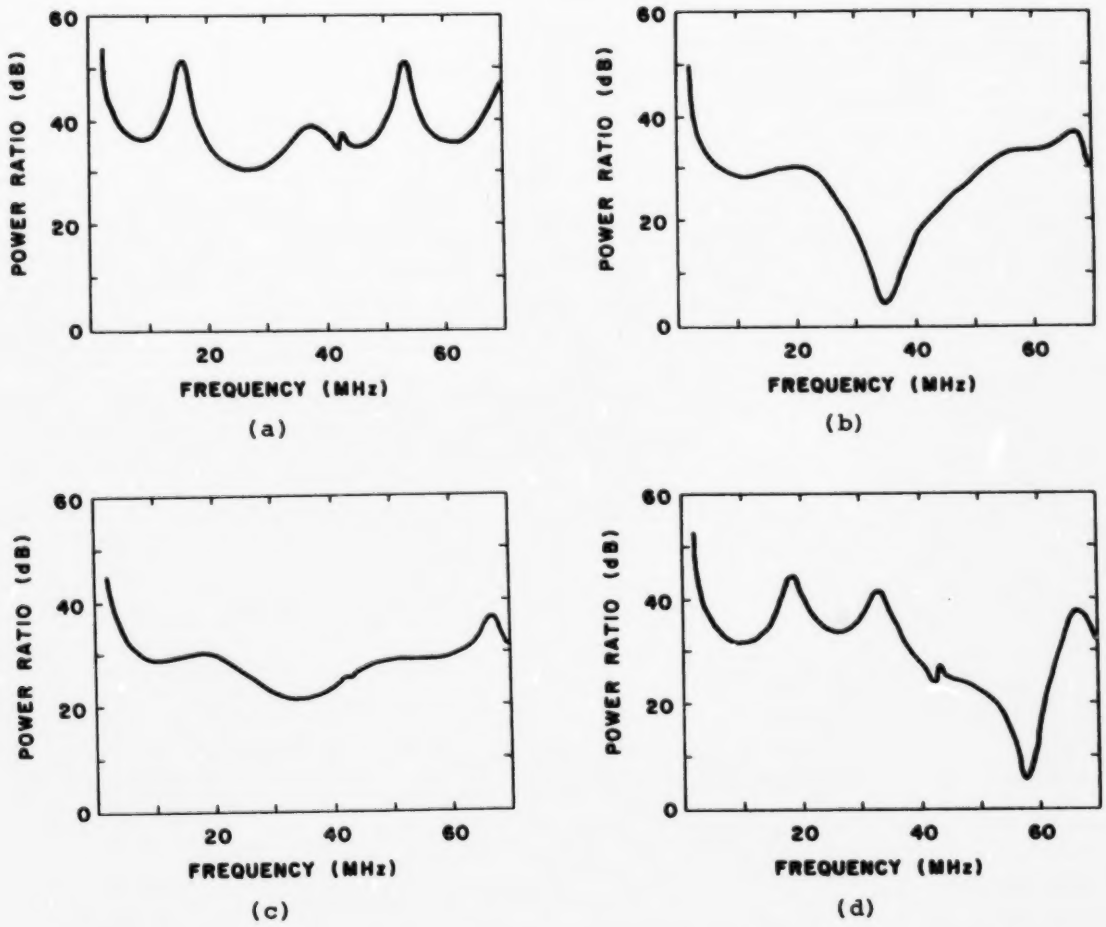

Figure 11. 1 m × 0.25 m × 0.25 m box in TEM cell: (a) empty cell; (b) grounded box; (c) box terminated in 50 Ω; (d) box terminated in open circuit.

This method readily indicates the frequencies at which the measurements will not be valid when an EUT is introduced into the cell. Unfortunately, it does not indicate the voltage on the center conductor opposite the EUT for the same reasons that reflectometer measurements will not.

MONITOR PROBE

The long axis of the 20-cm monopole monitor probe is parallel to the X-axis of the cell and lies in the unused half of the cell directly opposite the EUT. It is capacitor coupled to the Monitor Buffer Amplifier which

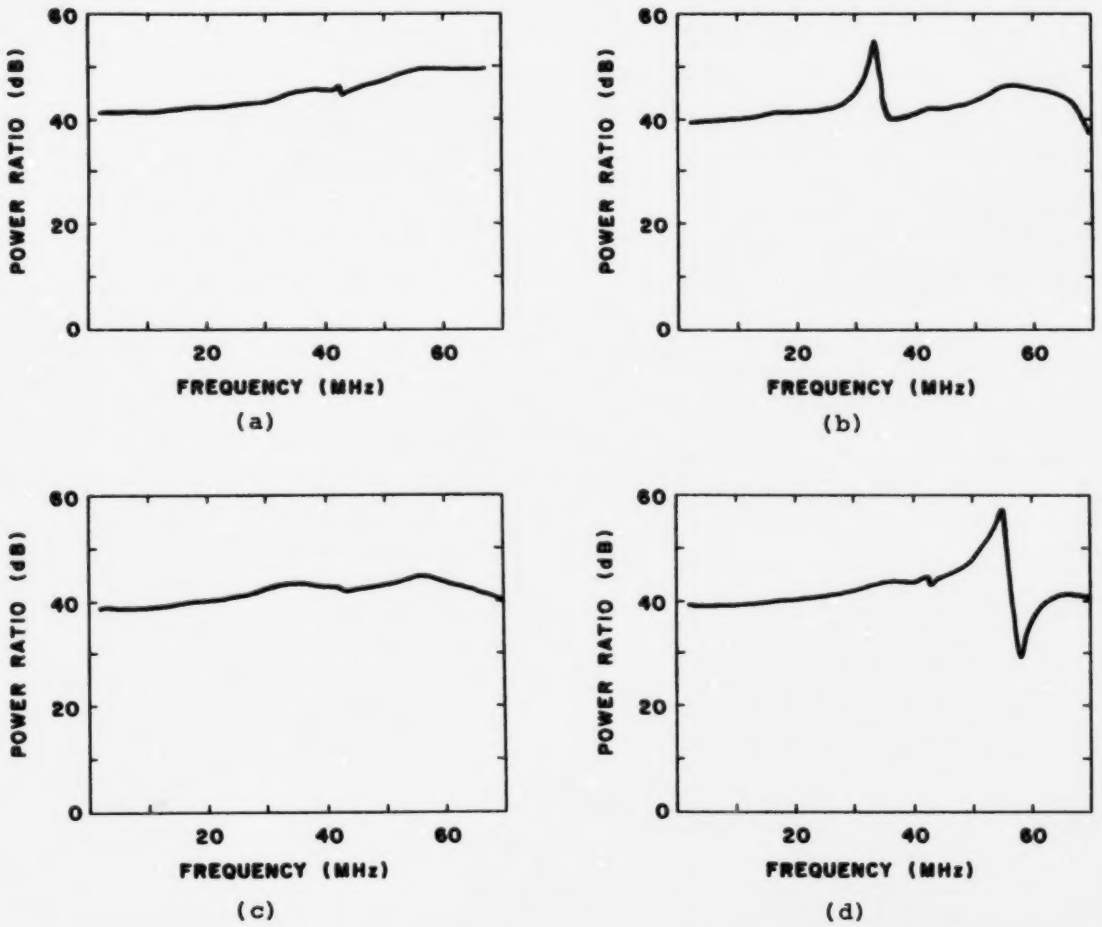

Figure 12. TEM cell monitor probe output, 1 m × 0.25 m × 0.25 m box: (a) empty cell, (b) grounded box, (c) box terminated in 50 Ω, (d) box terminated in open circuit.

converts the varying input impedance of the monopole to a 50-Ω output impedance over the frequency range of interest. The results of measurements made in a similar fashion to the previous sections are shown in Fig. 12. The signal input of the Spectrum Analyzer was connected to the Monitor Amplifier. The ratio of the power input to the Cell to the power output of the Monitor Amplifer is plotted on the Y-axes of Fig. 12. It is seen, in Fig. 12a, that the response of the probe over the frequency range is linear with a rise of about 9 dB from 2 to 70 MHz. The linearity would indicate that the probe was sufficiently small so as not to influence the voltage on the center conductor. The rise in the power ratio (loss of output) is due to the frequency response of the integrated circuit used in the buffer amplifier.

When the box is inserted in the cell, it is seen that reactions are produced at the same frequencies as Fig. 11. What is surprising is that the voltage on the center conductor both increases and decreases at these frequencies, relative to the values for the empty cell. This variation in the voltage opposite the EUT of the center conductor could not be predicted from Fig. 11. Therefore, if an EUT with its longest dimension greater than 46 cm is to be measured in the cell, the Monitor Probe should be used to determine the voltage on the center conductor.

SUSCEPTIBILITY MEASUREMENTS

The initial equipment calibration should be repeated at least once a week or prior to any measurements if the system has been idle for longer than a week. Initially, the power meter is calibrated using its internal calibrator. The rf input of the spectrum analyzer is then connected to the internal calibrator of the power meter.

The controls of the spectrum analyzer are then set for a sweep from 45 to 55 MHz, bandwidth of 300 kHz, with the input attenuator set at 20 dB, and the slow sweep connected to the interface unit. With an appropriate program, a recording is made. The input attenuator is adjusted to 30 dB and a second recording is made. A second HP85 program allows the determination of the maximum readings obtained on the spectrum analyzer. This permits the calibration of the readings of the interface of the signal level outputs of the spectrum analyzer in terms of dB/division and the dBm level of the minimum reading. Thus, the power and spectrum analyzer are calibrated using the same source.

The equipment is then set up as in Fig. 5. The frequency output of the spectrum analyzer is set at approximately 35 MHz, and the input voltage to the cell is adjusted by means of the switched attenuator to the required value. The controls of the spectrum analyzer are set for a sweep of 0.50 MHz/division, bandwidth of 300 kHz and the input attenuator at 10 dB. It is connected to the monitor buffer. A suitable program permits a recording of the monitor probes output over this frequency range. Midway through this program, the user is told to adjust the frequency range to 20 to 70 MHz. The power meter, spectrum analyzer level and the frequency counter values are then recorded for each of the 256 steps of the interface. This recording, A, is saved as it is the calibration of the monitor probe.

The EUT is then installed in the cell — usually on a wooden shelf at the center of the cell. If ac line power is required, it is available through a 150-Ω LISN installed in the wall of the cell. The EUT is usually

mounted about 10 cm from the wall of the cell. The line cord is run directly to the wall and then to the LISN with any additional cord taped the wall.

The same program as for calibrating the monitor probe is used and the same values recorded on tape B. As the program is stepping through the frequencies, any deviation in the behaviour of the EUT due to interference is noted by an observer by pressing the push button.

A second program initially takes the calibration tape A and for each frequency point determines the voltage on the center conductor from the power meter reading versus the output of the monitor probe at each frequency point. The output of the monitor probe is then read from the second recording B and converted to voltage on the center conductor using the calibration from A. This requires some interpolation because the frequency points of the two recordings are unlikely to be identical. The distance of the center of the EUT from the wall is measured and its multiplying factor is determined from Fig. 4. For example, if the center of the EUT was 1 m from the septum the multiplying factor would be 0.643. For convenience, in practice Fig. 4 is replotted with the distances measured from the wall. This multiplying factor is included in the program and a plot of the field strength versus frequency is produced as shown in Fig. 13. The frequencies at which the EUT reacted are shown by the breaks in the straight line at the 1-V level. Figure 13 shows the susceptibility of a battery operated multimeter. The meter was placed where the display was visible to an observer. The meter was set on the 2-V dc range with its test leads arranged in a loop approximately 9 in. in diameter. The leads were connected to an A cell. When the system was in operation, the observer pressed the push-button whenever the voltmeter reading differed by more than one digit from its normal reading of 1.41 V. The normal reading was obtained with the rf

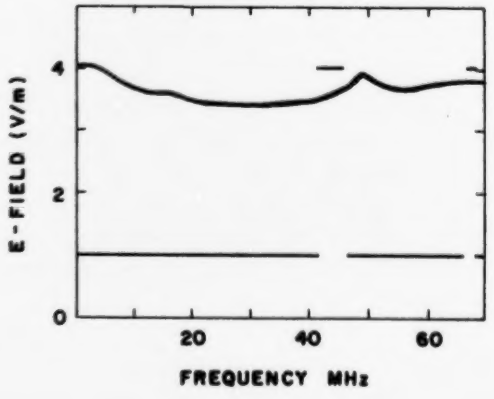

Figure 13. Susceptibility measurement of digital voltmeter, Radio Shack type 22-191, 2-V dc range.

power switched off to the cell. It is seen that the voltmeter reacted at frequencies of about 42 and 68 MHz. To determine exact susceptibility levels, it is easy to return to these frequencies and adjust the rf input level to the cell at each frequency in turn.

In practice, one would then rotate the EUT to a position where its axes would be mutually perpendicular and repeat measurement B. Similarly, a third position would be chosen and a third recording made. The susceptibility of the unit would be the worst case of the three positions measured.

ANTENNA FACTOR MEASUREMENTS

The determination of antenna factors for simple antennas (monopoles, dipoles, loops) follows much the same procedures as susceptibility measurements. The monitor probe is calibrated in the empty cell as in procedure A of the previous section. A frequency run is made with the antenna installed in the cell as in procedure B. Note that the long axis of a dipole or monopole antenna lies along the X axis of the cell. However, this is first processed in the same manner as the calibration run. If the two resulting curves do not differ substantially, which is the case for short antennas, a simplified procedure can be used. A frequency run is made with the spectrum analyzer input connected to the output of the EUT (run C). The power meter reading is converted to volts on the center conductor. This is multiplied in turn by a mulitplying factor determined from Fig. 4 to give the field strength. The input to the spectrum analyzer is converted to volts out of the antennas. Division of the field strength by the volts out of the antenna provides the antenna factor over the frequency range of the system.

For antennas which do react with the cell, the procedure involves an extra stage. By the same method as used in the susceptibility measurements, recordings A and B are combined to provide the field strength at the center of the antenna.

Run C provides the output voltage of the antenna. Again division provides the antenna factor. The method just described involves two frequency interpolations, which leads to some error, but the error in these more difficult cases is less at critical frequencies than that of the single run of the previous paragraph.

CONCLUSIONS

To determine the electromagnetic susceptibility of an electronic equipment or the antenna factor of a simple antenna in a TEM cell, the

field strength at the point where the equipment is situated must be known. It has been shown that the simplest and most accurate method is to measure the field strength in the unused half of the cell directly opposite the equipment with a small calibrated probe.

It is also known that this method should be used with all antennas and equipment for frequencies higher than one-tenth the upper useful frequency limit, i.e., 7.57 MHz. Below this frequency, measuring the voltage at the input to the cell probably provides sufficient accuracy.

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APPENDIX A: NUMERICAL DETERMINATION OF POTENTIAL AND ELECTRIC FIELDS

Vacuum tube designers have made extensive use of Laplace's Equations in determining the potential fields existing in vacuum tubes with multiple electrodes (1). The method employed is also known as the Relaxation or Iteration Method. Reference A1 illustrate the mathematical theory of the application and will not be repeated here. However, the practical application of the method and the expected errors will be illustrated in subsequent paragraphs.

Consider an infinitely long coaxial cable with 1 V on its center conductor relative to the zero voltage on its outside conductor. One quadrant of the cross section of the cable is divided into a convenient square grid as shown in Fig. A1. The potential at any of the points inside the cable can be accurately calculated from the following well-known expressions:

$$
\begin{aligned}
[A1] \qquad P &= 1, && 0 \leq r \leq 1 \\
&= 1 - \ln r/\ln 3, && 1 \leq r \leq 3 \\
&= 0, && r \geq 3
\end{aligned}
$$

The electric field intensity in the space between the inner and outer conductors is given by

$$[A2] \qquad E = -1/r \ln 3, \quad 1 \leq r \leq 3$$

To apply the Laplace Equation's in two dimensions, points C and D are initially assigned a value of 1 V while points A and E are assigned a value of 0. The points B, F, G, and H are assigned some initial value, say 0.5 V (the exact values of the intermediate points have no bearing on the final solution but do affect the number of iterations required for the final value).

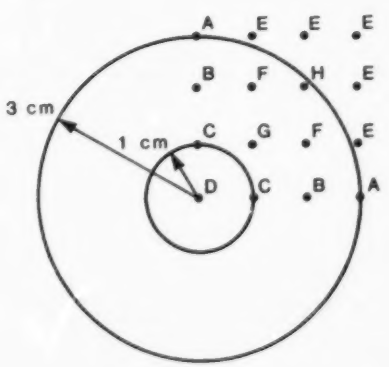

Figure A1. Cross section of a coaxial cable.

New values for B, F, G, and H are then calculated by taking the average of the four values for the points immediately surrounding it. For example,

[A3] $$P_{FNEW} = (P_B + P_H + P_E + P_G)/4$$

Note that only one quadrant of the coaxial cable has been considered. In this case, as with the Crawford cells considered in this report, there is symmetry which reduces significantly the number of calculations required. Points at the edge of the quadrant such as B are calculated by

[A4] $$P_{BNEW} = (2P_F + P_A + P_C)/4$$

Note that in a two-dimensional application such as this, the divisor is always 4.

After the new potentials at each point are calculated in turn, always using the newest values for each point, the process is repeated. This is continued until all the new values differ from their previous values by less than a preselected amount. The author used a value of 0.1%, which was convenient when using a computer.

The above procedure provides the potentials at the selected points. To obtain the electric fields at one of these points (say F) in the X direction, the difference between the two adjacent points is divided by the distance (2 cm), e.g.,

[A5] $$E_F = (P_B - P_H)/2 \quad \text{V/cm}$$

For a point on the edge of the inner or outer conductor (say C) the corresponding electric field is given by

[A6] $$E_C = P_B/1$$

Similarly the fields in the Y direction are calculated.

The total field at any point is calculated by

[A7] $$E_{total} = (E_x^2 + E_y^2)^{1/2}$$

With the 4 × 4 grid, the worst case deviation of the calculated value and the original value is 4.5 dB at point H. If the grid spacing is reduced by 1/2, i.e., 7 × 7, the maximum deviation is 2.2 dB at point G and with a 13 × 13 grid, 1.1 dB at point G. The latter cases require more computation time than the simpler 4 × 4 grid. It should also be pointed

out that transformation of coordinates will produce more accurate results for the coaxial cube as noted in reference A1. This example used here was to illustrate the fact that although the grid does not exactly match the geometry, useful results can be obtained.

Tippet and Chang (A2) show how the E fields in a TEM cell can be derived by means of multiple transformations. They provide an example of a 50 cm × 50 cm cell with a 41.23 septum. To compare results using the Laplace Equations, an 11 × 11 grid was fitted to one quadrant at the center of the cell. In order to fit the grid, the width of the septum was reduced to 40 cm with the knowledge that some errors would occur in the vicinity of the edge of the septum. After 100 iterations, the potential changes from one iteration to the next were less than 0.1%. Comparison of the total electric field intensities derived by this method and that of Tippet and Chang showed that they disagreed by less than 1.1 dB. As expected, the worst case occurred in the region of the edge of the septum.

Laplace's Equations can also be applied in the three-dimensional case. In deriving the new potential, the adjacent points on all three axes are summed and divided by six. In the case of a symmetrical Crawford cell, the number of iterations can be significantly reduced by fitting the cubical grid to one octant since the other octants will be mirror images. Similarly to the two-dimensional case, points along the mutual boundaries with other octants must be calculated using double values for some of the adjacent values, i.e., all new potentials must be calculated by summing six adjacent potentials and dividing the result by six.

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